PME7J004 ROBOTICS

(PROFESSIONAL ELECTIVE)

MODULE – I

1. Fundamentals of Robotics: Evolution of robots and robotics, Definition of industrial robot, Laws of Robotics, Classification, Robot Anatomy, Work volume and work envelope, Human arm characteristics, Design and control issues, Manipulation and control, Resolution; accuracy and repeatability, Robot configuration, Economic and social issues, Present and future application.
2. Mathematical modeling of a robot: Mapping between frames, Description of objects in space, Transformation of vectors.

Direct Kinematic model: Mechanical Structure and notations, Description of links and joints, Kinematic modeling of the manipulator, Denavit-Hartenberg Notation, Kinematic relationship between adjacent links, Manipulator Transformation matrix.

MODULE – II

3. Inverse Kinematics: Manipulator workspace, Solvable of inverse kinematic model, Manipulator Jacobian, Jacobian inverse, Jacobian singularity, Static analysis.
4. Dynamic modeling: Lagrangian mechanics, 2D- Dynamic model, Lagrange-Euler formulation, Newton-Euler formulation.
5. Robot Sensors: Internal and external sensors, force sensors, Thermocouples, Performance characteristic of a robot.

MODULE – III

6. Robot Actuators: Hydraulic and pneumatic actuators, Electrical actuators, Brushless permanent magnet DC motor, Servomotor, Stepper motor, Micro actuator, Micro gripper, Micro motor, Drive selection.
7. Trajectory Planning: Definition and planning tasks, Joint space planning, Cartesian space planning.
8. Applications of Robotics: Capabilities of robots, Material handling, Machine loading and unloading, Robot assembly, Inspection, Welding, Obstacle avoidance.

TEXT BOOKS:

1. Robotics and Control, R.K. Mittal and I.J. Nagrath, Tata McGraw Hill
2. Introduction to Robotics: Mechanics and control, John J Craig, PHI
3. Robotics Technology and Flexible Automation, S.R.Deb and S. Deb, TMH

REFERENCE BOOKS:

1. Introduction to Robotics, S. K. Saha, Tata McGraw Hill
2. Robotics: Control, Sensing, Vision and Intelligence, K.S.Fu, R.C.Gonzalez and C.S.G.Lee, McGraw Hill
3. Robotics, Appuu Kuttan K.K., I.K. international
4. Robot Dynamics and Control, M.W.Spong and M. Vidyasagar , Wiley India.
5. Industrial Robotics Technology, programming and application, M.P.Groover, TMH.
6. Introduction to Robotics: Analysis, Systems, Applications, S.B.Niku, PHI
7. Robotics: Fundamental Concepts and Analysis, A. Ghosal, Oxford University Press
8. Fundamentals of Robotics: Analysis and Control, R. J. Schilling, PHI
9. Robotic Engineering: An Integrated Approach, R.D. KLAFTER, T. A. Chmielewski, and M. Negin, PHI
10. Robot Technology: Fundamentals: J. G. Keramas, Cengage Learning

Czech spoke in
Czech Republic
European Union

Robot → Came from Czech word: Robota which means forced or slave laborer
→ In 1921 Karel Capek used a robot in Drama.
→ First commercial, Digital & Programmable robot was built by George Devol (1954).

→ According to International Organization for Standardization (ISO):-
An automatically controlled, reprogrammable, multipurpose manipulator programmable in three or more axes, which can be either fixed in place or mobile for use in Industrial automation applications.

→ According to Robot Institute of America (RIA):-
It is a reprogrammable multi-functional manipulator designed to move materials, parts, tools or specialized devices through variable programmed motions for the performance of variety of tasks.

Note:- A CNC machine is not a robot
→ CNC machine can do more than one task at a time but robot can do only one task at a time.
→ CNC machine have generally 2 or 3 degree of freedom but robot has 6 degree of freedom.
→ In CNC machine coding are done by G code and M code but in robot the method of programming is different.
→ CNC machine is more rigid in comparision with robot.

Robotics → It is a science, which deals with the issues related to design, manufacturing & usages of robot.
→ In robotics we try to copy 3H of humans
H: Hand → In form of manipulator (mech. hand)
H: Head → Arti. Intelligence.
H: Heart → Emotion

→ We use robots in manufacturing industries because to
- To Increase Productivity
- To Reduce Product cost
- Improve product Quality

→ Examples of robot:-

(i) ASIMO → ASIMO is a humanoid robot created by HONDA in 2000. It is currently displayed in Miraikan museum in Tokyo, Japan.

(Advanced Step Innovative Mobility)

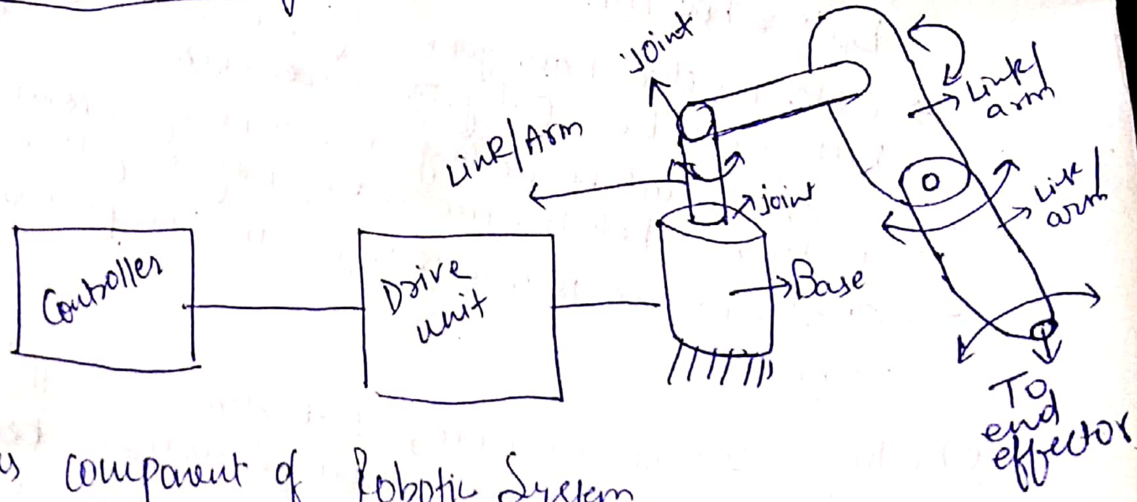
(ii) Sophia → Sophia is a social humanoid robot developed by Hong Kong based company Hanson Robotics in 2016. (14th Feb).. It can able to display more than 50 facial expressions.

Q) Write down 10 examples of robot which are used in different fields.

~~Q) How evolution of robot & robotics~~

Q) Write a short note on evolution of robot & robotics.

A Robotic System



Various component of Robotic System

→ Base → Link & joints → End effector/gripper

→ Wrist → Drive ~~Actuator~~ unit → Controller

→ Sensors.

→ Industrial Robot :-

- An Industrial robot is a robot system used for manufacturing. Industrial robots are automated, programmable and capable of movement on three or more axis.
- Typical applications of robots include welding, painting, assembly pick & place, packaging, product inspection & testing. All are done with high endurance, speed & precision.

Laws of Robot :-

Issac Asimov's Three Laws of Robotics :-

→ A robot may not harm the Human Being.

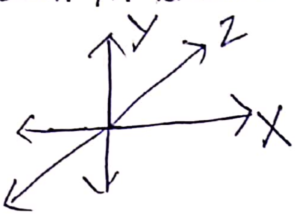
→ A robot must obey the order given by Human except it should not conflict with the 1st Law.

→ A robot must protect its own existence as long as it should not conflict with the 1st & 2nd Law.

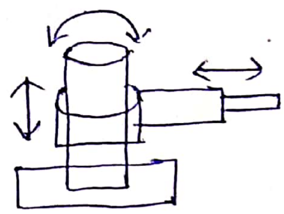
Classification of robots :-

I) According to Geometric classification :-

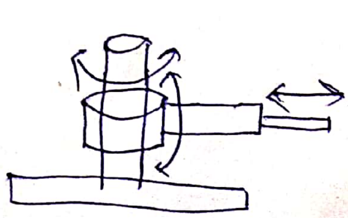
(i) Cartesian/Rectilinear: It moves in X, Y & Z direction. It has Three degree of freedom in linear motion only.



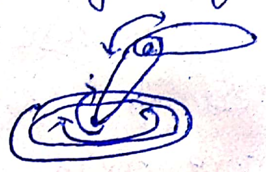
(ii) Cylindrical robot :- This type of robot is used for round workpiece.
It donot rotate the complete cycle.
→ Work on 2 linear and 1 rotory motion.



(iii) Spherical robot :- This type of robot is also used for round workpiece where cylindrical robots are not used.
→ It consist of 2 rotory & 1 linear motion.



(iv) Joined Arm robot :- This type of robot only perform rotory motion. This type of robot have 3 rotory motion.



(v) SCARA robot :- SCARA Robot has 3 rotory motion & 1 linear motion. It is mainly use for pick & place of any object.

II) According to the usage in different fields.

(i) Industrial Robot :→ Used in welding, material handling, productivity imp...
Ex→ Robolab, Lego etc.

(ii) Educational Robot → Used in education.

(iii) Domestic robot :→ used in home such as modern programmed toys which can talk, Dance etc.

Robot Anatomy :-

3.8 Work space or Work volume

The work volume is the term that refers to the space within which the robot can manipulate its wrist end. The convention of using the wrist end to define the robot's work volume is adopted to avoid the complication of different sizes of the end effectors that might be attached to the robot's wrist. The end effector is an addition to the basic robot should not be counted as the part of the robot work space. Also, the end effector attached to the wrist might not be capable of reaching certain points within the robot's normal work volume because of the particular combination of joint limits of the arm.

The work volume is determined by the following physical characteristics of the robot:

- The robots physical configuration (types of joints, structure of links)
- The size of the body, arm and wrist components.
- The limit of the robots joint movements.

3.9 CONCLUDING REMARK

This chapter provided the basic

Nearly all industrial robots have mechanical joints that can be classified into following five types as shown in Figure 7.5.2.

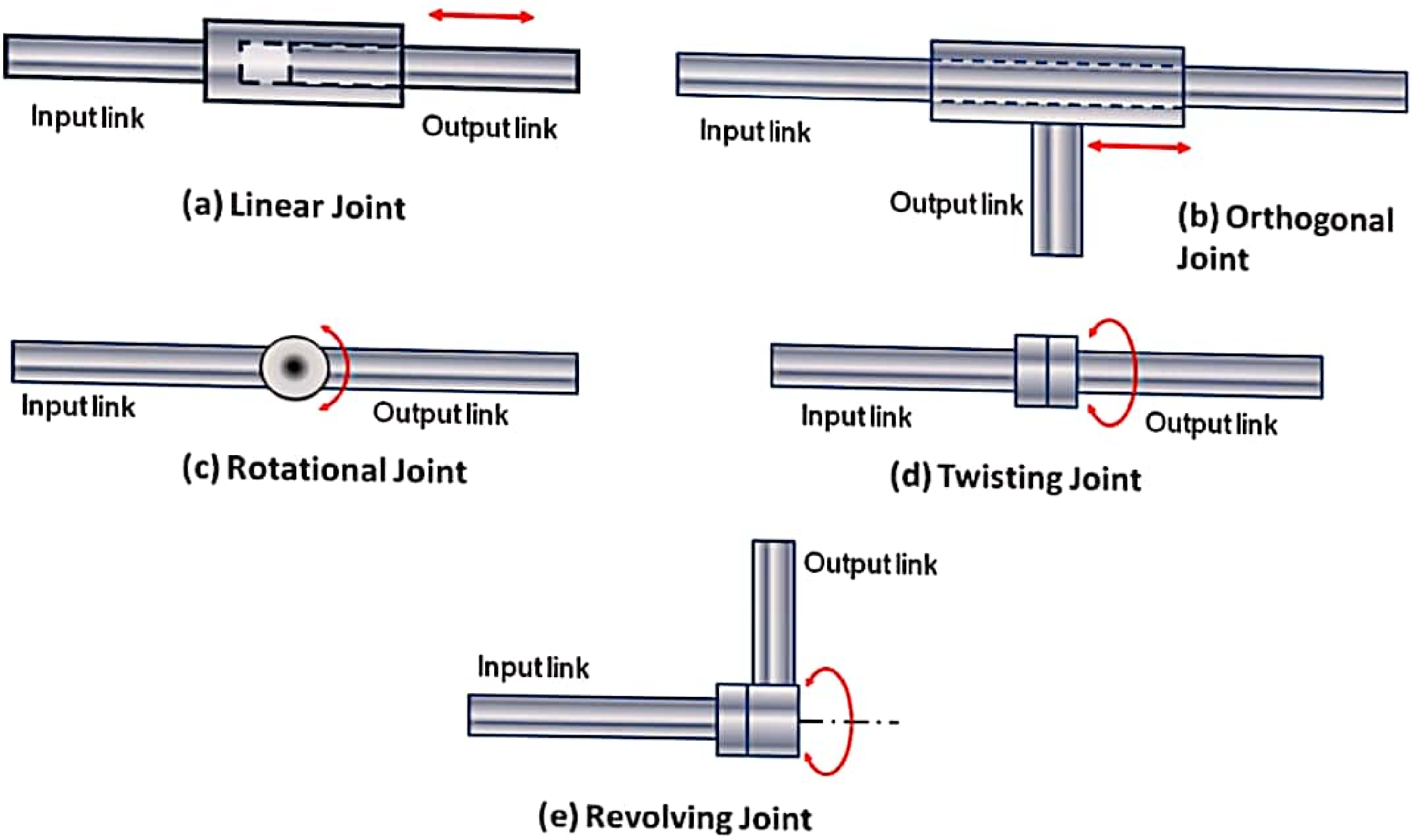


Fig. 7.5.2 Types of Joints

a) Linear joint (type L joint)

The relative movement between the input link and the output link is a translational sliding motion, with the axes of the two links being parallel.

b) Orthogonal joint (type U joint)

This is also a translational sliding motion, but the input and output links are perpendicular to each other during the move.

c) Rotational joint (type R joint)

This type provides rotational relative motion, with the axis of rotation perpendicular to the axes of the input and output links.

d) Twisting joint (type T joint)

This joint also involves rotary motion, but the axis or rotation is parallel to the axes of the two links.

e) Revolving joint (type V-joint, V from the "v" in revolving)

In this type, axis of input link is parallel to the axis of rotation of the joint. However the axis of the output link is perpendicular to the axis of rotation.

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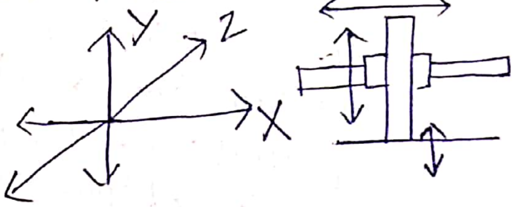
→ It consist of 3 Prismatic joint or linear joint.

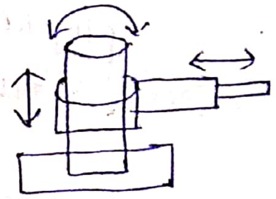


(ii) Cylindrical robot :- This type of robot is used for round workpiece.

It donot rotate the complete cycle.

→ Work on 2 linear and 1 rotory motor.

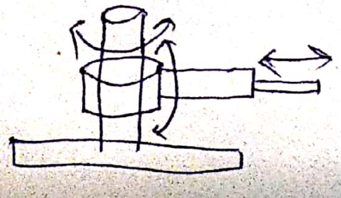
→ It consist of 2 P & 1 R joint.



(iii) Spherical robot :- This type of robot is also used for round workpiece where cylindrical robots are not used.

→ It consist of 2 rotory & 1 linear motion.

→ It consist of 2 R & 1 P



(iv) Joined Arm robot :- This type of robot only perform rotory motion. This type of robot have 3 rotory motion.

→ It consist of 3 R joint.

(v) SCARA robot :- SCARA Robot has 3 rotory motion & 1 linear motion. It is mainly use for pick & place of any object.

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~~# Robot Anatomy :-~~

Work Envelope & Work Volume :-

The Work Volume is the term that refers to the space within which the robot can manipulate its wrist end. The end effector is an addition to the basic robot should not be counted as the part of the robot work space. Also, the end effector attached to the wrist might not be capable of reaching certain points within the robot's normal volume because of the particular combination of joint limits of the arm.

The work volume is determined by the following physical characteristics of the robot:-

→ The robot physical configuration

→ The size of the body, arm & wrist component.

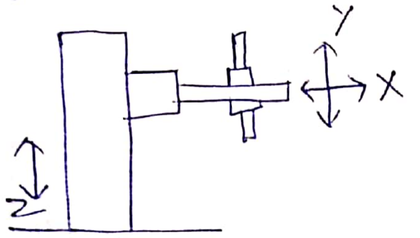
→ The limit of the robots joint movements.

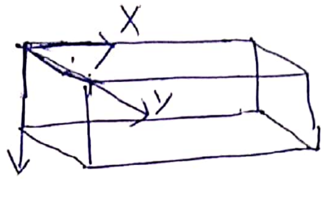
Work Volume & Work Envelop

The four basic configuration of robot are:-

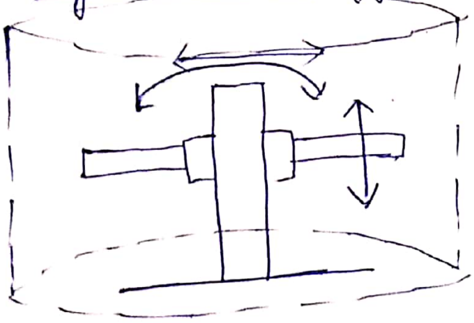
(i) Cartesian (Rectangular) configuration:- All 3 Prismatic joint

It is constructed by three ⊥r slide giving only linear motion in 3 ⊥r principal axis. End point of the ⊥r arm is capable to operate in a cuboidal space called workspace for cartesian configuration.



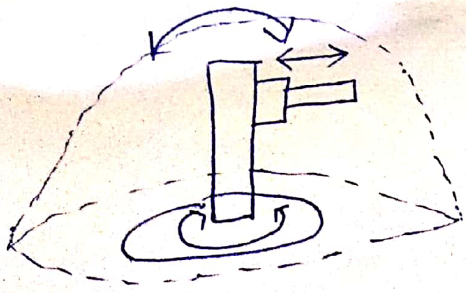


(ii) Cylindrical Configuration:-



It consist of 2 ⊥r Prismatic joint & a revolute joint. The arm end is thus capable to sweep cylindrical space. The work space of cylindrical configuration is hollow cylinder.

(iii) Spherical or Polar configuration:-



It consist of 1 Prismatic joint & 2 rotary joint.
In this 1 Prismatic joint and 1 Revolute joint are mounted on revolute base, which give capability to move the arm end point within a partial spherical cell as work volume.

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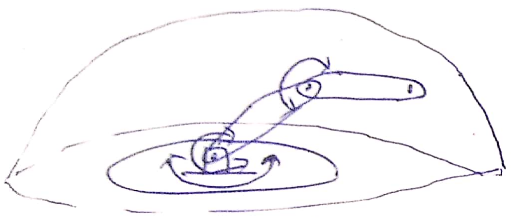
(iv) Articulate (Revolute or joined arm) configuration:-



The articulated arm is the best simulates of human arm and manipulator. It consist of two straight link with rotary joint which is similar to the human arm. The work volume of articulate configuration is spherical shape.

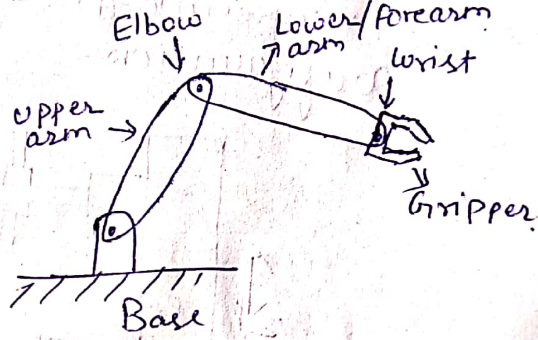
Human Arm Characteristics:

The human arm & its capacity make the human separate from other animals. The design of human arm structure is unique and difficult to replicate. Robot manipulator follow the design of human arm, which is similar to the following motion of hand.

$-90 \leq$ Pitch ≤ 50

$-45 \leq$ Yaw $\leq 15^{0}$

$-180 \leq$ Roll ≤ 90



→ Robotic arms are used in industries where humans are not able to work i.e, high temperature, polluted air zone, heavy weight lifting etc. Robotic arm also used for perform a task accurately in various environment.

→ To provide the roll motion to the hand, forearm, and the upper arm both undergo a twist, while pitch & yaw are provided by the wrist joint. The 2-DOF shoulder joint provides an approximately hemispherical sweep.

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Design & Control issue :-

→ Robots are design to perform more & more variety of highly skilled jobs with minimum human assistance.

→ The mechanical structure of robot consist of joint, link etc has poor stiffness, accuracy.

→ This problem can overcome by advanced design & control technique

→ The position & motion of each joint is affected by the position & motion of other joint.

→ The weight & inertial load of each link is carried by its previous link.

→ The link undergo rotary motion about the joint making centrifugal & coriolis effect.

→ All this make the dynamic behaviour of robot manipulator complex and non-linear.

→ The kinematic & dynamic complexities create control problem and control system design is a challenging task.

→ The environment in which robots are used faces more complexicity as compare to other conventional machine tool.

→ Robots are work in very complex environment i.e, high temperature, polluted air zone, heavy weight lifting etc.

→ Robot also have to perform additional task i.e, planning, inspection, quality control etc.

→ The control strategies are coded in a form so that robot can execute accurately.

→ For storing data, command memory are needed.

→ The programming & command generation become critical issue in robotics.

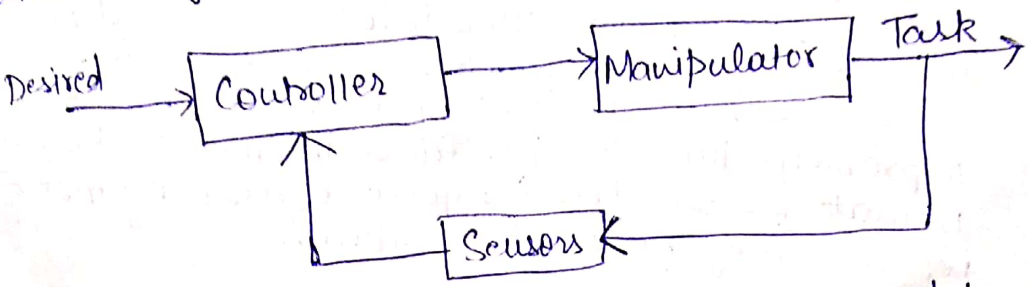
→ To monitor its arm motion disturbance & unpredictable environment robot require interfacing with internal & external sensor.

→ Various sensors are also used in robot to perform various task accurately.

Manipulation & Control:-

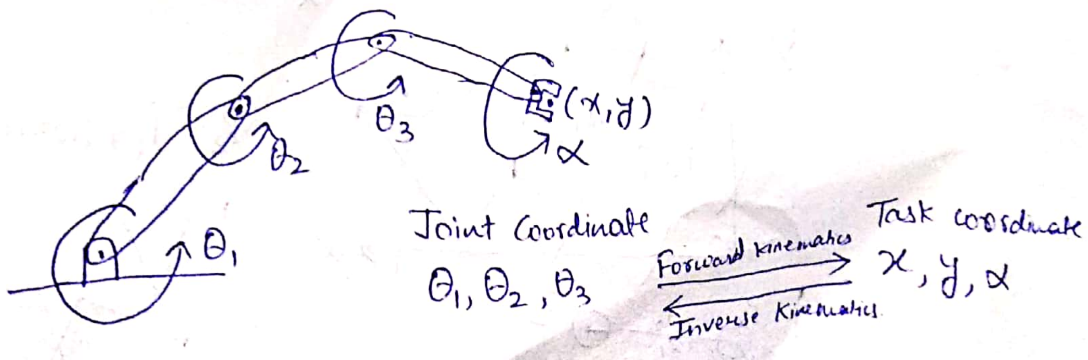
The tasks to be performed by the manipulator are:
(i) to move the end-effector along a desired trajectory.
(ii) to exert a force on the environment to carry out the desired task.

The controller of manipulator has to control both task i.e, position & force. A schematic sketch of a typical controller is given in following figure.

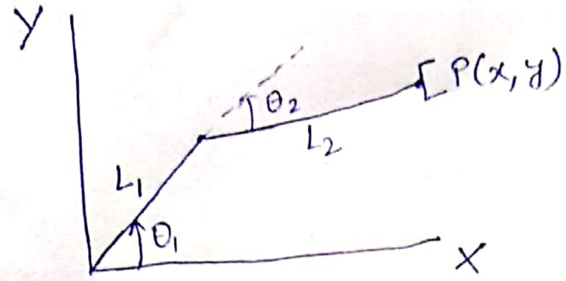


The positions, velocity, forces and torques are measured by sensors and based on these measurements and the desired behaviour, the controller determines the input so that the end effector carries out the desired task as closely as possible.

→ To describe position & orientation of a body in space, a frame is attached to the body. The position & orientation of this frame with respect to some reference coordinate frame called base frame. Frames are attached to joints, links, end effectors and workpiece in the environment.



→ Consider the simplest two link problem as shown in figure.

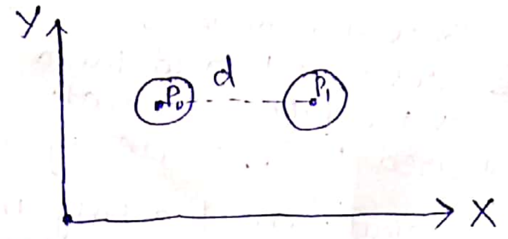


There are two links L_1, L_2 and assume that the joint angles are θ_1 & θ_2 and the coordinates of end effector point P is (x, y). From simple geometry analysis from this manipulator, it is possible to compute coordinates (x, y) from the joint angle θ_1 & θ_2.

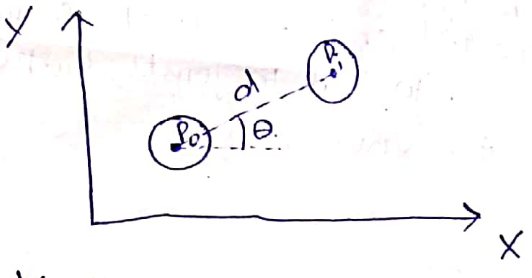
The basic problem in the study of mechanical manipulation is to calculate the position of the end effector when the joint angles are known. This is called forward kinematics. The inverse kinematics problem is to determine the joint angles when the position of end effector is given.

A problem that can be faced in inverse kinematics is that the solution for joint angles may not be unique i.e., there may be multiple solution.

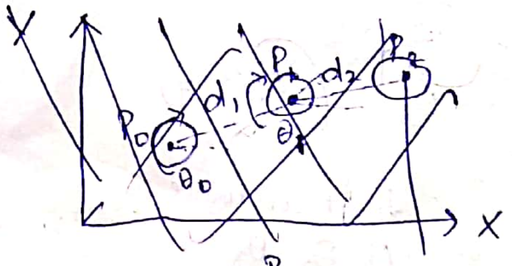
Ex→



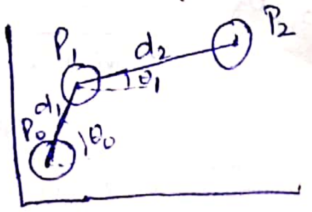
$$P_1 = P_0 + d$$



$$P_1 = P_0 + \text{rotate}(d, P_0, \theta)$$



$$P_2 = P_1 + \text{rotate}(d_2, P_1, \theta_0 + \theta_1)$$



$$P_n = P_{n-1} + \text{rotate}\left(d_n, P_{n-1}, \sum_{k=0}^{n-1} \theta_k\right)$$

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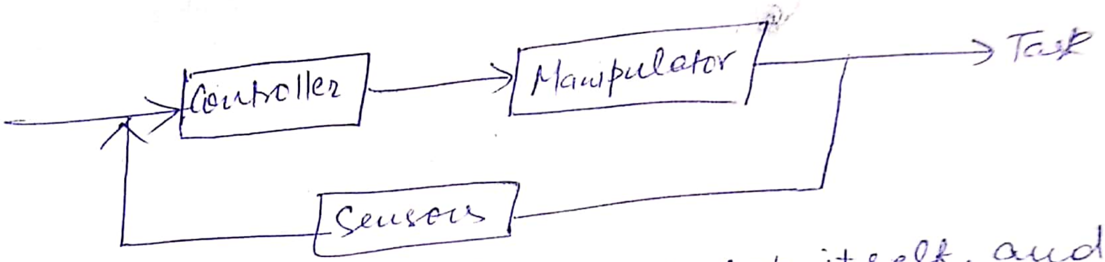
Sensors

There are generally two categories of sensors used in robots. These are sensors for internal purposes and for external purposes. Internal sensors are used to monitor and control the various joints of the robot. They form a feedback control loop with the robot controller.



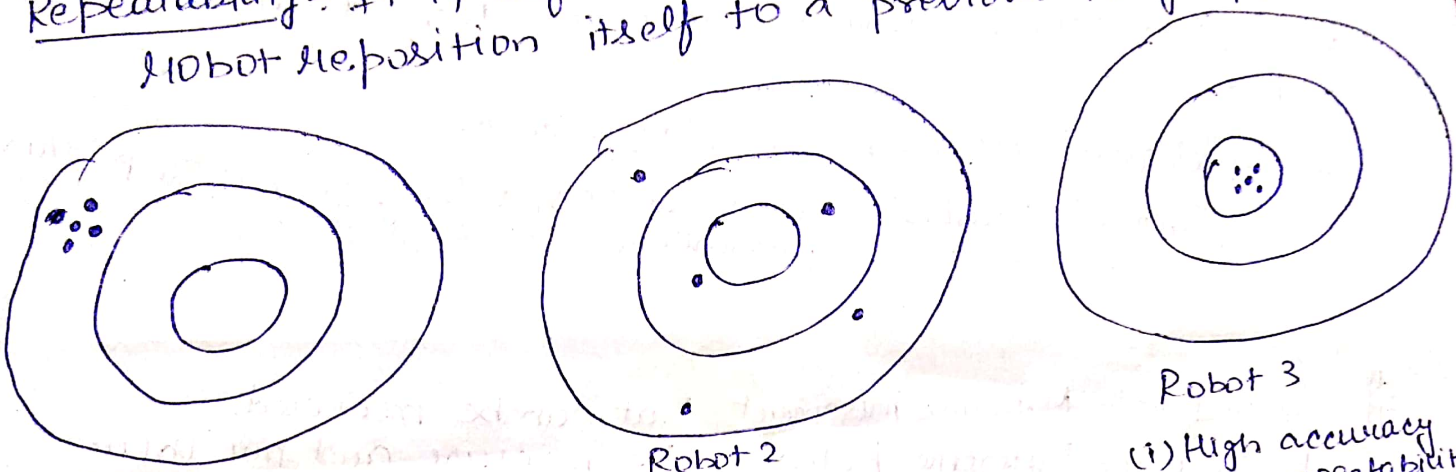
External sensors are external to the robot itself, and are used when we wish to control the operation of robot.

Ex- External sensors:- Force, Torque sensor

Internal sensors:- Position sensors, Velocity sensors, Temperature, pressure, Joint sensor, Acc^n sensor.

Accuracy :- It is the precision with which a computed point can be reached

Repeatability :- It is defined as the precision with which a robot reposition itself to a previous target point.



Robot 1
(i) High Repeatability
Low Accuracy

Robot 2
(i) Low Accuracy
Low Repeatability

Robot 3
(i) High accuracy
High repeatability.

Resolution

It is defined as the smallest allowable position increment of a robot that can be measured.

Resolution

- Programming resolution
  Smallest allowable position increment in robot program.
- Control Resolution
  Smallest change in position that the feedback device can be measure.

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Economical & Social Issue:-

1. Economic consideration:-

(a) Element of cost:-

Before Introduction of robot in a workplace comparision of cost is require i.e, capital cost vs the benifit to be gained.
The cost include in robot installation consist of

→ **Capital cost:-**

Capital cost consist of hardware, software, human support & cost of robot.
Factors such as physical size, load capacity, control capability, accuracy, repeatability influence the capital cost.

→ **Cost of end effector**

End effector transform the programme in to useful task For performing specific work, specialised end effector are required and costing may depend upon design, development manufacturing & testing. The end effector cost consist of significant portion of robot cost.

→ **Cost of support equipment:-**

Support equipments for transportation, orientation, position are required. It can be carried either by human beings or specialised equipments.

→ **Sensor feedback:-**

Sensor feed back is necessary to perform any task. By the help of sensor feedback robot can do any task accurately.

→ **The cost of safety.**

Safety is required by isolating the area of robot in which they are working during difficult task such as carry of workpiece having high temperature, high weight etc.

→ Cost of Installation

For introduction of robot in any industry installation cost is require according to their specification.

→ Cost of maintenance:-

→ Cost of programming.

B) → Cost of staff training.

2. Social Consideration:-

(i) Integration of robot in the work place:-
Integration of robot in the work place increase the productivity but reduce the working man due to which workers became jobless.

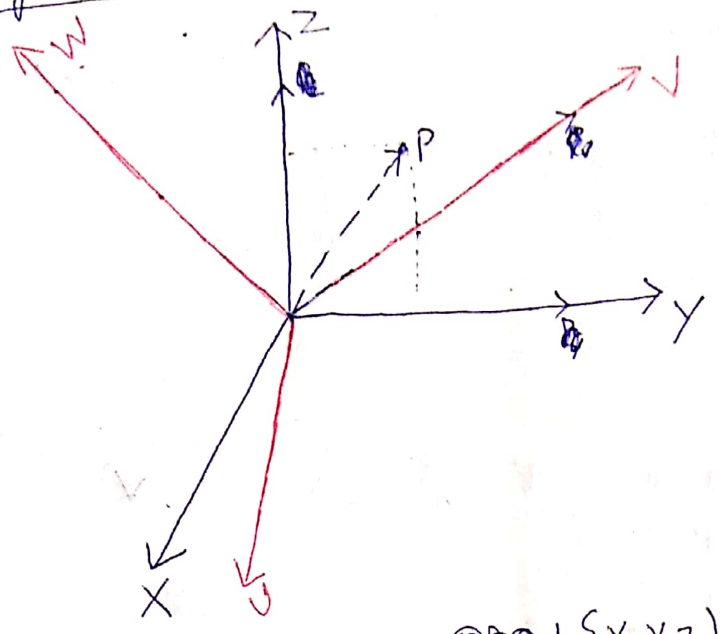
(ii) Robot & Management:-
By use of robot in industries ensure that customer need are met according to the specification, quality, quantity etc. By use of robot competitive position in market also meat.

(iii) Robot & workforce:-
Robot can lift the workpiece having high weight due to which unemployablity increases. There are many reason that robot can replace humans:-
→ Perform boring, repetative & mindless task.
→ Carry out heavy & hazardous task.
→ work in toxic environment.
→ Reduce the pressure & stress to meet the target.

(iv) Robot in manufacturing:-
Robots are used for manufacturing with high reliability, controlability & accuracy as compare with humans. By use of robots any product are manufactured by highly technical skills.

(v) The problem of robot sensing:-
Robot know holding, sensing with physical sensor and can decide that the work should be done or not. but humans can decide that the work should be done or not by seeing.

- ## Mapping between Rotated Frames



Consider there are two frames 1{X, Y, Z) & 2(U, V, W). A point P described by the two frames and can be expressed by

$${}^1P = {}^1P_x \hat{x} + {}^1P_y \hat{y} + {}^1P_z \hat{z} \quad \text{—— (I)}$$

$${}^2P = {}^2P_u \hat{u} + {}^2P_v \hat{v} + {}^2P_w \hat{w}. \quad \text{—— (II)}$$

The description of point P in frame 2 is known and its description in frame 1 is to be found.

$${}^1P_x = {}^2P \cdot \hat{x} = \hat{x} \cdot ({}^2P_u \cdot \hat{u} + {}^2P_v \hat{v} + {}^2P_w \hat{w})$$

$${}^1P_y = {}^2P \cdot \hat{y} = \hat{y} \cdot ({}^2P_u \hat{u} + {}^2P_v \hat{v} + {}^2P_w \hat{w})$$

$${}^1P_z = {}^2P \cdot \hat{z} = \hat{z} ({}^2P_u \hat{u} + {}^2P_v \hat{v} + {}^2P_w \hat{w})$$

So,

$${}^1P_x = {}^2P_u \cdot \hat{x} \cdot \hat{u} + {}^2P_v \hat{x} \hat{v} + {}^2P_w \hat{x} \hat{w}$$

$${}^1P_y = {}^2P_u \hat{y} \hat{u} + {}^2P_v \hat{y} \hat{v} + {}^2P_w \hat{y} \hat{w}$$

$${}^1P_z = {}^2P_u \hat{z} \hat{u} + {}^2P_v \hat{z} \hat{v} + {}^2P_w \hat{z} \hat{w}$$

Now, write it in matrix form.

$$\begin{bmatrix} {}^1P_x \\ {}^1P_y \\ {}^1P_z \end{bmatrix} = \begin{bmatrix} \hat{x} \cdot \hat{u} & \hat{x} \hat{v} & \hat{x} \hat{w} \\ \hat{y} \hat{u} & \hat{y} \hat{v} & \hat{y} \hat{w} \\ \hat{z} \hat{u} & \hat{z} \hat{v} & \hat{z} \hat{w} \end{bmatrix} \begin{bmatrix} {}^2P_u \\ {}^2P_v \\ {}^2P_w \end{bmatrix}$$

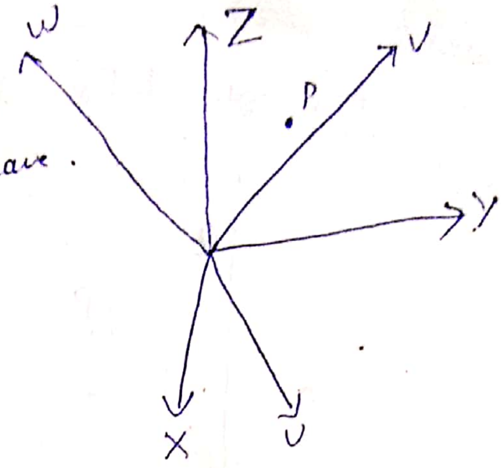
$${}^1P = {}^1R_2 \, {}^2P \quad \text{—— (III)}$$

where,

$${}^1R_2 = \begin{bmatrix} \hat{x} \hat{u} & \hat{x} \hat{v} & \hat{x} \hat{w} \\ \hat{y} \hat{u} & \hat{y} \hat{v} & \hat{y} \hat{w} \\ \hat{z} \hat{u} & \hat{z} \hat{v} & \hat{z} \hat{w} \end{bmatrix} \quad \text{—— (IV)}$$

$^1P = {}^1P_x\hat{x} + {}^1P_y\hat{y} + {}^1P_z\hat{z}$

$^2P = {}^2P_u\hat{u} + {}^2P_v\hat{v} + {}^2P_w\hat{w}$



Discription of Point P is known in ^1P & we have to find the discription of Point P in frame 2

$^2P_u = {}^1P \cdot \hat{u} = {}^1P_x \hat{u}\cdot\hat{x} + {}^1P_y \hat{y}\hat{u} + {}^1P_z \hat{u}\hat{z}$

$^2P_v = {}^1P \cdot \hat{v} = {}^1P_x \hat{v}\hat{x} + {}^1P_y \hat{v}\hat{y} + {}^1P_z \hat{v}\hat{z}$

$^2P_w = {}^1P \cdot \hat{w} = {}^1P_x \hat{w}\hat{x} + {}^1P_y \hat{w}\hat{y} + {}^1P_z \hat{w}\hat{z}$

$$\begin{bmatrix} P_u \\ P_v \\ P_w \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{u}\hat{x} & \hat{u}\hat{y} & \hat{u}\hat{z} \\ \hat{v}\hat{x} & \hat{v}\hat{y} & \hat{v}\hat{z} \\ \hat{w}\hat{x} & \hat{w}\hat{y} & \hat{w}\hat{z} \end{bmatrix}}_{^2R_1} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

Similarly, when we find the discription of point P in frame 2 when discription of point P in frame 1 is known. Then, $\boxed{^2P = {}^2R_1 {}^1P}$ — (V)

$$^2R_1 = \begin{bmatrix} \hat{u}\hat{x} & \hat{u}\hat{y} & \hat{u}\hat{z} \\ \hat{v}\hat{x} & \hat{v}\hat{y} & \hat{v}\hat{z} \\ \hat{w}\hat{x} & \hat{w}\hat{y} & \hat{w}\hat{z} \end{bmatrix} \quad \text{— (VI)}$$

From (IV) & (VI)

$$^1R_2 = [^2R_1]^T \quad \text{— (VII)}$$

From (III) & (VII)

$$^1P = [^2R_1]^T\, {}^2P \quad \text{— (VIII)}$$

From (V)

$$^1P = {}^2P[^2R_1]^{-1} \quad \text{— (IX)}$$

From (VIII) & (IX)

$$^1P = [^2R_1]^T\, {}^2P = {}^2P[^2R_1]^{-1}$$

$$\Rightarrow [^2R_1]^T = [^2R_1]^{-1}$$

$$\Rightarrow [^2R_1^T][^2R_1] = I$$

$$\Rightarrow \boxed{R R^T = I}$$

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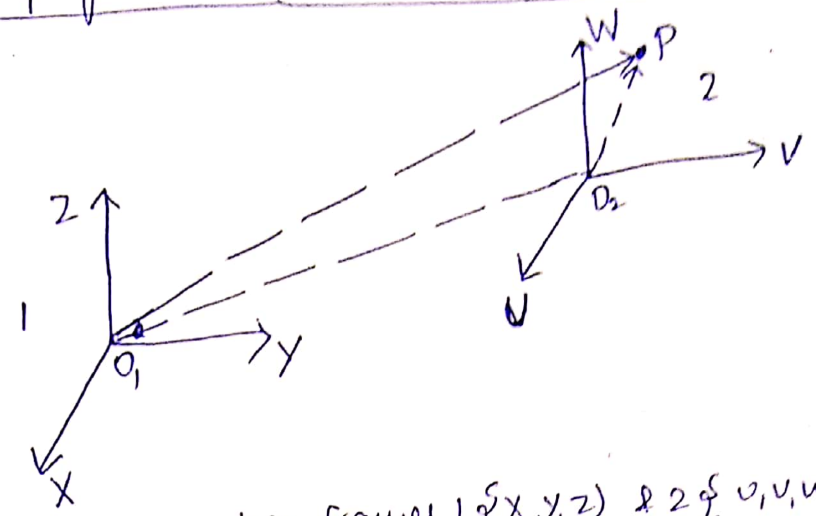
Maping between Translated frames



Consider there are two frames 1 {x, y, z} & 2 {u, v, w} having origin point O_1 & O_2 respectively. In this frame 2 moves in translation motion with respect 1.

In this vectors can be written as.

$$\vec{O_1P} = \vec{O_2P} + \vec{O_1O_2}$$

$$\Rightarrow {}^1P = {}^2P + {}^1D_2$$

Now,

$${}^1P = ({}^2P_u + d_x)\hat{x} + ({}^2P_v + d_y)\hat{y} + ({}^2P_w + d_z)\hat{z} \quad \text{— (I)}$$

$${}^1P = {}^1P_x\hat{x} + {}^1P_y\hat{y} + {}^1P_z\hat{z} \quad \text{— (II)}$$

comparing (I) & (II)

$${}^1P_x = {}^2P_u + d_x$$

$${}^1P_y = {}^2P_v + d_y$$

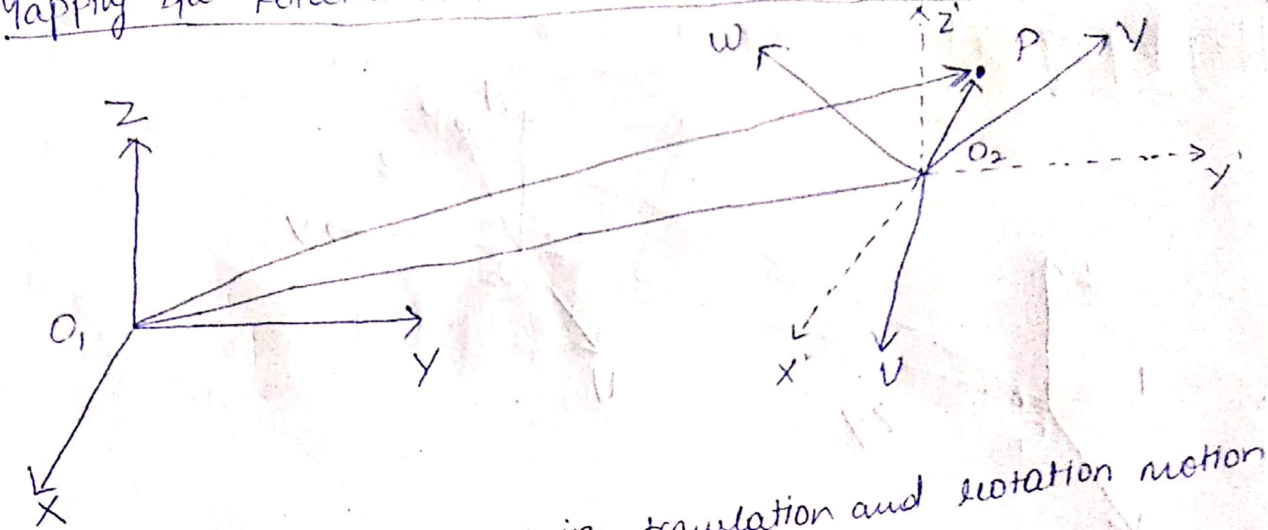
$${}^1P_z = {}^2P_z + d_z$$

We can write equation (I) in the form of matrix

$${}^1P = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^2P_u \\ {}^2P_v \\ {}^2P_w \\ 1 \end{bmatrix}$$

$${}^1P = {}^1T_2\,{}^2P$$

Mapping b/w Rotated & Translated ~~motion~~ frames



In this frame 2 moves in translation and rotation motion with respect to 1.

So, we can write as

$${}^{1}P = {}^{1}R_2\,{}^{2}P + {}^{1}D_2$$

$\therefore$

$${}^{1}P_x = {}^{2}P_u\,\hat{x}\hat{u} + {}^{2}P_v\,\hat{x}\hat{v} + {}^{2}P_w\,\hat{x}\hat{w} + d_x$$

$${}^{1}P_y = {}^{2}P_u\,\hat{y}\hat{u} + {}^{2}P_v\,\hat{y}\hat{v} + {}^{2}P_w\,\hat{y}\hat{w} + d_y$$

$${}^{1}P_z = {}^{2}P_u\,\hat{z}\hat{u} + {}^{2}P_v\,\hat{z}\hat{v} + {}^{2}P_w\,\hat{z}\hat{w} + d_z$$

$${}^{1}P = \left[\begin{array}{ccc|c} \hat{x}\hat{u} & \hat{x}\hat{v} & \hat{x}\hat{w} & d_x \\ \hat{y}\hat{u} & \hat{y}\hat{v} & \hat{y}\hat{w} & d_y \\ \hat{z}\hat{u} & \hat{z}\hat{v} & \hat{z}\hat{w} & d_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} {}^{2}P_u \\ {}^{2}P_v \\ {}^{2}P_w \\ 1 \end{bmatrix}$$

→ Rotation matrix (upper-left 3×3 block)

⇒ Translation matrix (d_x, d_y, d_z column)

The 1 at the bottom right → Scaling factor

So, we can write.

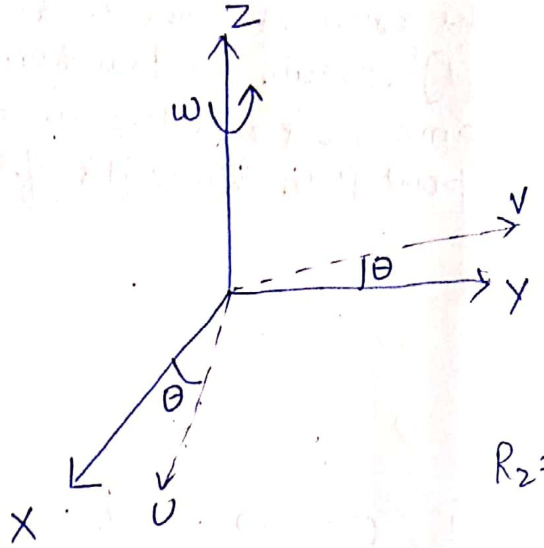
$${}^{1}T_2 = \left[\begin{array}{c|c} {}^{1}R_2 & {}^{1}D_2 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

So, $\boxed{{}^{1}P = {}^{1}T_2\,{}^{2}P}$

$\vec{a} \cdot \vec{b} = ab(\cos\theta)$

$$R = \begin{bmatrix} \hat{x}\hat{u} & \hat{x}\hat{v} & \hat{x}\hat{w} \\ \hat{y}\hat{u} & \hat{y}\hat{v} & \hat{y}\hat{w} \\ \hat{z}\hat{u} & \hat{z}\hat{v} & \hat{z}\hat{w} \end{bmatrix}$$

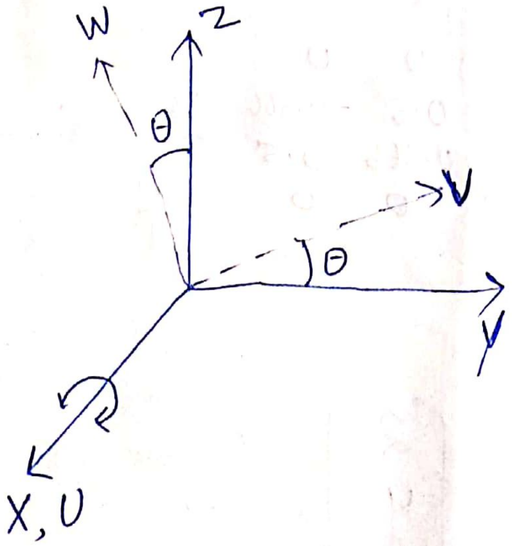
R_z



$$\begin{bmatrix} \cos\theta & \cos(90+\theta) & \cos 90^\circ \\ \cos(90-\theta) & \cos\theta & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

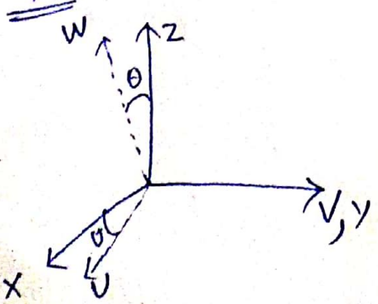
R_x



$$R_x = \begin{bmatrix} \cos 0 & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos\theta & \cos(90+\theta) \\ \cos 90^\circ & \cos(90-\theta) & \cos\theta \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

R_z



$$R_y = \begin{bmatrix} \cos\theta & \cos 90^\circ & \cos(90-\theta) \\ \cos 90^\circ & \cos 0 & \cos 90^\circ \\ \cos(90+\theta) & \cos 90^\circ & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix for single frame:

Rotation of frame with respect to z axis.

$$R(z) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation of frame with respect to x-axis

$$R(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation of frame with respect to y axis.

$$R(y) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Translation matrix (x, y) = ~~$\begin{bmatrix} T_x & 0 & 0 \\ 0 & T_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$~~ $\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}$

$$(X, z) = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & 0 \\ 0 & 0 & T_z \end{bmatrix}$$

Q) Using the transformation matrix, translate the point (2, 6) by 4 in x- direction and by -3 in the y-direction.

Ans. Translation matrix in X-Y direction:

$$T = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}_{3\times 1}$$

$$= \begin{bmatrix} 2+4 \\ 6-3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$$

So, after translation new point is (6, 3).

Q) Using a transformation matrix, rotate the point (2, 6) by 90° degree anticlockwise about (0, 0) along Z axis.

Ans.

$$R = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 2 \\ 1 \end{bmatrix}$$

New point is (-6, 2)

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Q) The coordinates of point P in frame {1} are $[3\ 2\ 1]^T$. The position vector P is rotated about the z-axis by 45°. Find the coordinates of point Q, the new position of point P.

Ans

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times0.707 - 2\times0.707 \\ 3\times0.707 + 2\times0.707 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 3.535 \\ 1 \end{bmatrix}$$

New point, $Q = [0.707\ \ 3.535\ \ 1]^T$

$$\begin{bmatrix} -6+4 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 \end{bmatrix}_{2\times2}\begin{bmatrix} 3 & 0 \end{bmatrix}_{2\times2} \quad \begin{bmatrix} 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \end{bmatrix}$$

Q) Frame 1 & 2 have coincident origins and differ only in orientation. Frame 2 is initially coincident with frame 1. Certain rotations are carried out about the axis of the fixed frame 1 i.e, first rotation about X-axis by 45° then about Y-axis by 30° and finally about X-axis by 60°. Obtain the rotation matrix and also find the new point for (2, 4, 6).

Ans.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ \\ 0 & \sin 60^\circ & \cos 60^\circ \end{bmatrix} \begin{bmatrix} \cos 30 & 0 & \sin 30^\circ \\ 0 & 1 & 0 \\ -\sin 30^\circ & 0 & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix} \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.707 & -0.707 \\ 0 & 0.707 & 0.707 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix} \begin{bmatrix} 0.866 & 0.3535 & 0.3535 \\ 0 & 0.707 & -0.707 \\ -0.5 & 0.612 & 0.612 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & 0.3535 & 0.3535 \\ 0.433 & 0.5\times0.707 - 0.866\times0.612 & -0.5\times0.707 - 0.866\times0.612 \\ -0.25 & 0.866\times0.707 + 0.5\times0.612 & -0.866\times0.707 + 0.5\times0.612 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & 0.3535 & 0.3535 \\ 0.433 & -0.17 & -0.883 \\ -0.25 & 0.918 & -0.306 \end{bmatrix}$$

- Inversion of Transformation matrix

If frame 2 is moving w.r.t frame 1

$$^{1}T_2 = \left[\begin{array}{c|c} ^{1}R_2 & ^{1}D_2 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

In this we have to find $^{2}T_1$ when $^{1}R_2$ & $^{1}D_2$ are known.

We know that

$$^{1}P = {^{1}R_2}\,{^{2}P} + {^{1}D_2}$$

Multiply both side by $^{2}R_1$

$$^{2}R_1\,{^{1}P} = {^{2}R_1}\,{^{1}R_2}\,{^{2}P} + {^{2}R_1}\,{^{1}D_2}$$

$$^{2}R_1\,{^{1}P} = {^{2}P} + {^{2}R_1}\,{^{1}D_2}$$

$$^{2}P = {^{2}R_1}\,{^{1}P} - {^{2}R_1}\,{^{1}D_2} \quad \text{—(I)}$$

We also know that

$$^{2}P = {^{2}R_1}\,{^{1}P} + {^{2}D_1} \quad \text{—(II)}$$

from (I) & (II)

$$^{2}R_1\,{^{1}P} - {^{2}R_1}\,{^{1}D_2} = {^{2}R_1}\,{^{1}P} + {^{2}D_1}$$

$$^{2}D_1 = -{^{2}R_1}\,{^{1}D_2}$$

$$^{2}D_1 = -[^{1}R_2]^T\,{^{1}D_2}$$

$$^{2}T_1 = \left[\begin{array}{c|c} ^{2}R_1 & ^{2}D_1 \\ \hline 0\ 0\ 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} [^{1}R_2]^T & -[^{1}R_2]^T\,{^{1}D_2} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Q) Frame {2} is rotated with respect to frame {1} about the x-axis by an angle of 60°. The position of the origin of frame 2 as seen from frame {1} is ${}^1D_2 = [7 \quad 5 \quad 7]^T$. Obtain the transformation matrix ${}^1T_2$, which describes frame {2} relative to frame {1}. Also, find the description of point P in frame {1} if ${}^2P = [2 \quad 4 \quad 6]^T$

Ans.

$$ {}^1T_2 = \begin{bmatrix} {}^1R_2 & {}^1D_2 \\ 0\ 0\ 0 & 1 \end{bmatrix} $$

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & \cos\theta & -\sin\theta & 5 \\ 0 & \sin\theta & \cos\theta & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & \cos 60^\circ & -\sin 60^\circ & 5 \\ 0 & \sin 60^\circ & \cos 60^\circ & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Description of Point P

$$ = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0.5 & -.866 & 5 \\ 0 & 0.866 & 0.5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 2+7 \\ 4\times0.5-6\times0.866+5 \\ 0.866\times4+0.5\times6+7 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 1.804 \\ 13.464 \\ 1 \end{bmatrix} $$

Q) In the above example $^1T_2$ was obtained, which describes the position & orientation of frame 2 relative to frame 1. Using this matrix determine the description of frame 1 relative to frame 2.

Ans.

$$^1T_2 = \begin{bmatrix} ^1R_2 & ^1D_2 \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_1 = \begin{bmatrix} ^2R_1 & ^2D_1 \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} [^1R_2]^T & -[^1R_2]^T\,{}^1D_2 \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

$$^2R_1 = [^1R_2]^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.866 \\ 0 & -0.866 & 0.5 \end{bmatrix}$$

$$^2D_1 = -[^1R_2]^T\,{}^1D_2 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.866 \\ 0 & -0.866 & 0.5 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 7 \end{bmatrix}$$

$$= -\begin{bmatrix} 7 \\ 8.562 \\ -0.83 \end{bmatrix} = \begin{bmatrix} -7 \\ -8.562 \\ 0.83 \end{bmatrix}$$

So,

$$^2T_1 = \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 0.5 & 0.866 & -8.562 \\ 0 & -0.866 & 0.5 & 0.83 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q) Consider a point P in space. Determine the new location of the point (2,4,6) after rotating it by an angle of 45° about z-axis and then translating it by -1 unit along x-axis and -2 unit along z-axis. Find the new point location.

Ans.

$$T = \begin{bmatrix} & R & & D \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~$$T = \begin{bmatrix} \cos 45^\circ & 0 & -\sin 45^\circ & -1 \\ 0 & 1 & 0 & 0 \\ [illegible] & 0 & [illegible] & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$~~

$$T = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & -1 \\ \sin 45^\circ & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{New point} = \begin{bmatrix} 0.707 & -0.707 & 0 & -1 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 2\times0.707 - 4\times0.707 - 1 \\ 2\times0.707 + 4\times0.707 \\ 6-2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -2.414 \\ 4.242 \\ 4 \\ 1 \end{bmatrix}$$

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Mechanical Structure & Notation

→ A manipulator consist of a chain of rigid bodies called links, connected to each other by joints, which allow linear or revolute motion between connected links.

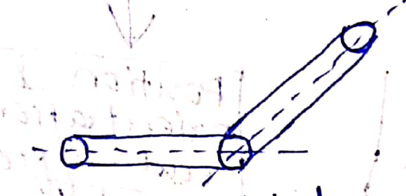
→ A joints with m degree of freedom can be modeled as m joints with one degree of freedom each connected with (m-1) links.

→ Most ~~Each link is connected to two other links without the formation of closed loops.~~

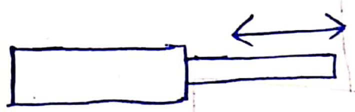
→ Most industrial robotic manipulator are open kinematic chains in which each link in connected to two other links without the formation of closed loops. In this all joints are motorized.

→ Two joints which are mainly used in industrial robots are (i) Revolute joints (ii) Prismatic joints.

→ Revolute joints allows rotational motion between connected links and Prismatic joint or sliding joint permits translation motion between the connected links.

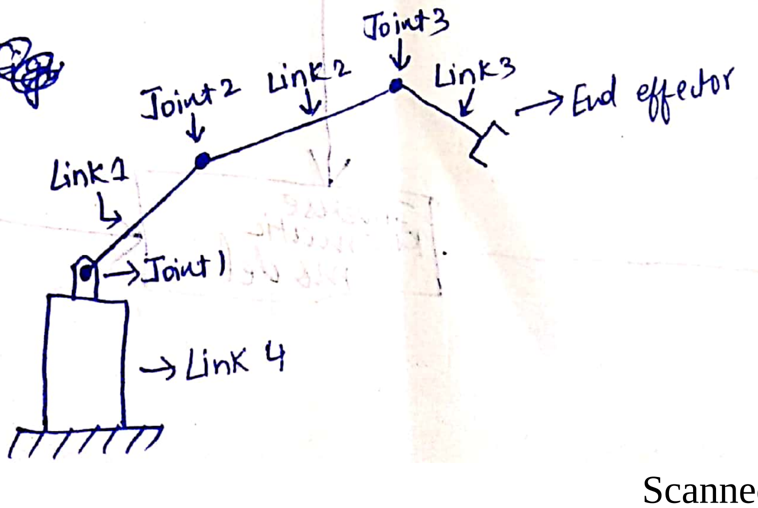


Revolute joint



Prismatic or sliding joint.

→



→ Above figure is of 3-DOF manipulator arm in an open kinematic chain having 3 revolute joint & 4 links. So, n-DOF manipulator arm consists of (n+1) link & n joints.

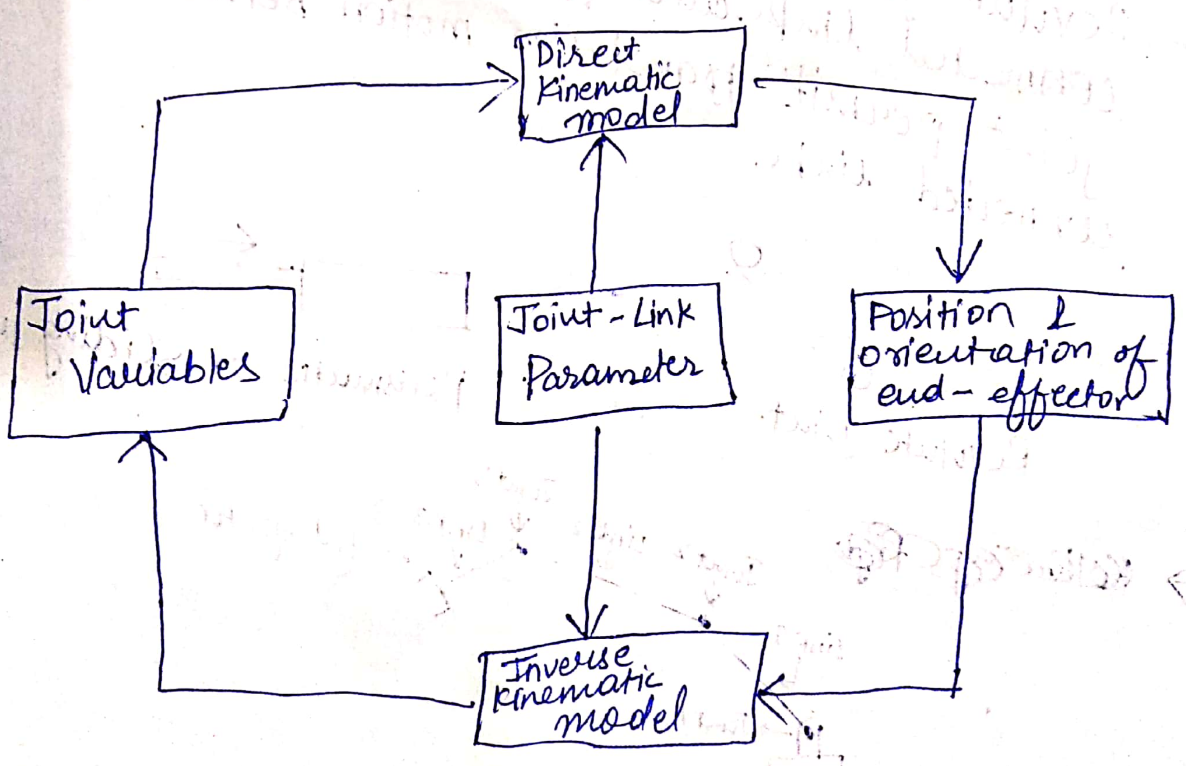
~~Description of links & joints~~

Kinematic Modelling of the manipulator

The kinematic modeling problem is split into two problems are:-

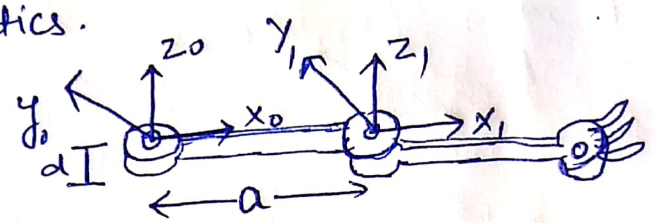
(i) Given the set of joint-link parameters, the problem of finding the position & orientation of the end-effector with respect to a known reference frame. This is referred as Direct kinematic model.

(ii) For a given position & orientation of the end-effector, it is required to find a set of joint variables that would bring the end-effector in the specified position & orientation. This is referred as inverse kinematic model...



Denavit - Hartenberg Notation:-

→ This Convention links the homogeneous transformation (A_i) with the four basic transformations to find out its forward Kinematics.



θ = A rotational angle between two links, about the z-axis i.e, b/w links.

d = The distance (offset) on the z-axis

a = The length of each link.

α = The twist angle.

$$A_i = Rot_z \theta_i \; Trans_z d_i \; Trans_x a_i \; Rot_x \alpha_i$$

$$A_i = Rot_{z}\,\theta_i \cdot Trans_{z}\,d_i \; Trans_{x}\,a_i \; Rot_{x}\,\alpha_i$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, a_i = link length

α_i = link twist

d_i = link offset.

θ_i = Joint angle.

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i & 0 & a_i\sin\theta_i \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

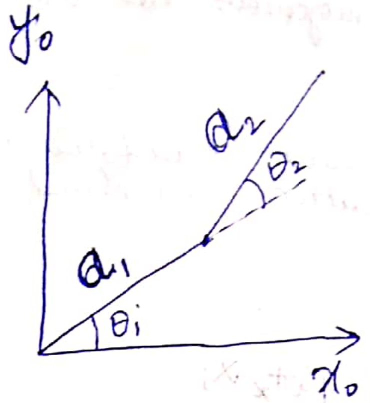
$\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$

$\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$

$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

$\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$

Ex →



Link	a_i	d_i	θ_i	α_i
1	a_1	0	θ_1	0
2	a_2	0	θ_2	0

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fixed Position → $T_1 = A_1$

$$T_2 = A_1 A_2$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 & 0 & a_2\cos\theta_1\cos\theta_2 - a_2\sin\theta_1\sin\theta_2 + a_1\cos\theta_1 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & 0 & a_2\cos\theta_2\sin\theta_1 + a_2\sin\theta_2\cos\theta_1 + a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & a_2\cos(\theta_1+\theta_2) + a_1\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & a_2\sin(\theta_1+\theta_2) + a_1\cos\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$x = a_2\cos(\theta_1+\theta_2) + a_1\cos\theta_1$

$y = a_2\sin(\theta_1+\theta_2) + a_1\sin\theta_1$

Q) Find the Position of the tool for following data

Link	θ_i	d_i	a_i	α_i
1	120°	0	0	90°
2	0	200 mm	0	0

Ans

$$A_1 = \begin{bmatrix} \cos 120° & -\sin 120° \cdot \cos 90° & \sin 120° \cdot \sin 90° & 0 \\ \sin 120° & \cos 120° \cos 90° & -\sin 90° \cos 120° & 0 \\ 0 & \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 0 & 0.867 & 0 \\ 0.867 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos 0 & -\sin 0 \cos 0 & \sin 0 \cos 0 & 0 \cos 0 \\ \sin 0 & \cos 0 \cdot \cos 0 & -\sin 0 \cos 0 & 0 \\ 0 & \sin 0 & \cos 0 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -0.5 & 0 & 0.867 & 0 \\ 0.867 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0.867 & 173.4 \\ 0.867 & 0 & 0.5 & 100 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$